

Econometrics II

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Lecture Structure

- ① t-test
- ② Testing multiple hypotheses

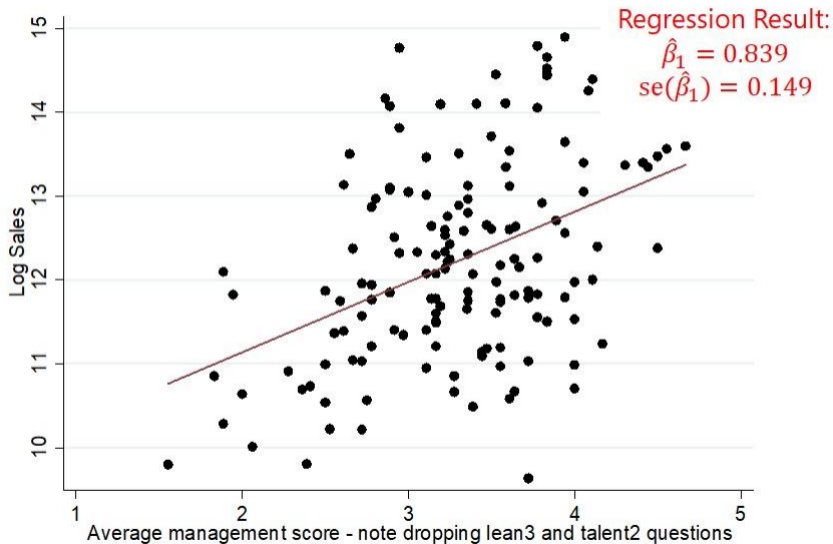
Recap from Last Lecture

- Projection matrix: $\mathbf{M}_X = \mathbf{I}_N - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$
- Estimator for σ^2 : $\hat{\sigma}^2 = \frac{RSS}{(N-k)}$
- GM- Theorem: Under GM1-GM4 OLS is BLUE
- t-test: $\frac{\hat{\beta} - \beta}{\sqrt{\hat{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}}} \sim t(N - k)$

Example t-Test

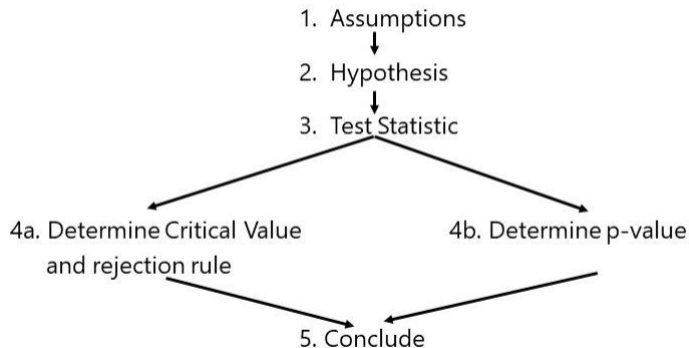
- The following example comes from the World Management Survey conducted by Nick Bloom (Stanford), John Van Reenen (MIT) and coauthors
 - They have developed a questionnaire to measure management quality in firms
 - We relate sales y to the average management score x
 - Focus on German sample (clearest relationship): $N = 148$

Scatter Plot and Regression Line



Perform t-Test

- We can now perform a t-test whether this relationship is significantly different from 0
- Tests always follow the following five steps:



Perform t-Test

- ① GM assumptions satisfied
- ② Hypotheses: $H_0 : \beta_2 = 0$; $H_1 : \beta_2 \neq 0$
- ③ Test statistic: $\frac{\hat{\beta}_2 - \beta_2^0}{se(\hat{\beta}_2)} = \frac{(0.839 - 0)}{0.149} = 5.63$ (as we have shown above this is distributed as a t)
- ④ Critical value from the t-tables (here 148-2 degrees of freedom): $t(100) = 1.984$
- ⑤ \rightarrow reject H_0

Testing Hypotheses Involving Multiple β s

- We have shown how to test hypotheses about a single β
- In some cases we want to test hypotheses about multiple β s
- Examples of linear hypotheses:
 - $H_0: \beta_2 = 5.2$
 - $H_0: \beta_1 - \beta_3 = 0$
 - $H_0: \beta_1 - \beta_3 = 0 \text{ and } \beta_4 = 6$
 - $H_0: \beta_j = 0$ for all j

Testing Multiple Hypotheses

- In matrix notation we can write such hypotheses as follows:

$$H_0 : R \beta = q$$

$r \times k$ $k \times 1$ $r \times 1$

- where r is equal to the number of linear hypotheses

Examples Multiple Hypotheses

- $H_0: \beta_2 = 5.2$: (suppose that we estimated 5 β s)

- $H_0: \beta_1 - \beta_3 = 0$

Examples Multiple Hypotheses

- $H_0: \beta_1 - \beta_3 = 0 \text{ and } \beta_4 = 6$

- $H_0 : \beta_j = 0 \text{ for all } j$

Testing Multiple Hypotheses

- It is important that the hypotheses do not involve redundancies or inconsistencies
- Example for redundancies:

- Example inconsistencies:

- If one “cleans” the hypothesis of redundancies and inconsistencies: $\text{rank}(R) = r$

Test Statistic

- There are 3 equivalent ways of testing such multiple hypotheses
- Here we discuss the procedure that requires the estimation of the unrestricted model and the restricted models (i.e. taken H_0 for granted and plugging in the restriction e.g. $H_0: \beta_1 - \beta_3 = 0$)
- We then get two RSS :
 - RSS_R the sum of squared residuals of the restricted model
 - RSS_U the sum of squared residuals of the unrestricted model
- Which RSS will be higher?
- We can then calculate the following test statistic:

F-Distribution

